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Two q-supercongruences from Jackson's ${}_6\phi_5$ summation

by
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Abstract

Employing Jackson's ${}_6\phi_5$ summation and the 'creative microscoping' method introduced by Guo and Zudilin, we prove two q-supercongruences modulo the cube of a cyclotomic polynomial. As conclusions, we obtain two supercongruences, one of which can be stated as follows: for any prime $p \equiv 1 \pmod 4$ and $0 \leqslant s \leqslant (p-1)/4$, modulo p^3,

$$\sum_{k=s}^{(3p-3)/4}(8k+3)\frac{(\frac{3}{4})_{k-s}(\frac{3}{4})_{k+s}(\frac{3}{4})_k^2}{(k-s)!(k+s)!k!^2} \equiv (3p+4s)\frac{(\frac{3}{4})_s^2(\frac{3}{4})_{(3p-3)/4+s}(\frac{3}{2})_{(3p-3)/4-s}}{s!^2(1)_{(3p-3)/4+s}(\frac{3}{4})_{(3p-3)/4-s}},$$

where $(x)_n = x(x+1)\cdots(x+n-1)$ is the Pochhammer symbol.

Key Words: q-supercongruences, supercongruences, Jackson's ${}_6\phi_5$ summation, creative microscoping.
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1 Introduction

In 1997, Van Hamme [5] presented 13 remarkable supercongruences corresponding to Ramanujan or to Ramanujan-like formulas for $1/\pi$. For instance, the infinite series

$$\sum_{k=0}^{\infty}(8k+1)\frac{(\frac{1}{4})_k^4}{k!^4} = \frac{2\sqrt{2}}{\sqrt{\pi}\,\Gamma(\frac{3}{4})^2},$$

corresponds to the following supercongruence for truncated hypergeometric series: for any prime $p \equiv 1 \pmod 4$,

$$\sum_{k=0}^{(p-1)/4}(8k+1)\frac{(\frac{1}{4})_k^4}{k!^4} \equiv p\frac{\Gamma_p(\frac{1}{2})\Gamma_p(\frac{1}{4})}{\Gamma_p(\frac{3}{4})} \pmod{p^3}, \tag{1}$$

which is marked (G.2) in Van Hamme's list. Here and throughout the paper, $(a)_n = a(a+1)\cdots(a+n-1)$ is the Pochhammer symbol, and $\Gamma(x)$ stands for Morita's p-adic Gamma function [8]. He [6] and Swisher [9] independently proved that (1) is true modulo the stronger power p^4.

Recently, motivated by an extension of Van Hamme's (A.2) supercongruence given by Guo [2], Tang [10] gave a generalization of (1) as follows: for any prime $p\equiv 1 \pmod 4$ and $0\leqslant s\leqslant (p-1)/4$,

$$\sum_{k=s}^{(p-1)/4}(8k+1)\frac{(\frac{1}{4})_{k-s}(\frac{1}{4})_{k+s}(\frac{1}{4})_k^2}{(k-s)!(k+s)!k!^2}\equiv (p+4s)\frac{(\frac{1}{4})_s^2(\frac{1}{4})_{(p-1)/4+s}(\frac{1}{2})_{(p-1)/4-s}}{s!^2(1)_{(p-1)/4+s}(\frac{1}{4})_{(p-1)/4-s}} \pmod{p^3}. \quad (2)$$

In fact, Tang proved (2) by establishing the following q-supercongruence: for any integer $n>1$ with $n\equiv 1 \pmod 4$ and nonnegative integer $s\leqslant (n-1)/4$,

$$\begin{aligned}\sum_{k=s}^{(n-1)/4}&[8k+1]\frac{(q;q^4)_{k-s}(q;q^4)_{k+s}(q;q^4)_k^2}{(q^4;q^4)_{k-s}(q^4;q^4)_{k+s}(q^4;q^4)_k^2}q^{2k}\\ &\equiv [n+4s]\frac{(q;q^4)_s^2(q;q^4)_{(n-1)/4+s}(q^2;q^4)_{(n-1)/4-s}}{(q^4;q^4)_s^2(q^4;q^4)_{(n-1)/4+s}(q;q^4)_{(n-1)/4-s}}q^{3s+(1-n)/4} \pmod{\Phi_n(q)^3}.\end{aligned} \quad (3)$$

Here we need to familiarize ourselves with the standard q-notation. The *q-integer* is defined by $[n]=1+q+\cdots+q^{n-1}$, the *q-shifted factorial* is defined as

$$(a;q)_n=\begin{cases}(1-a)(1-aq)\cdots(1-aq^{n-1}), & \text{if } n\geq 1,\\ 1, & \text{if } n=0,\end{cases}$$

and the *n-th cyclotomic polynomial* in q is given by

$$\Phi_n(q)=\prod_{\substack{1\leqslant k\leqslant n\\ \gcd(k,n)=1}}(q-\zeta^k),$$

where ζ is an n-th primitive root of unity.

It is clear that (2) follows from (3) by taking $n=p$ and $q\to 1$. Note that the $s=0$ case of (3) was given by Liu and Wang [7], which may also be derived from [3, Theorem 4.3].

In this paper, we shall establish the following q-supercongruence similar to (3).

Theorem 1.1. *Let $n\equiv 1 \pmod 4$ be a positive integer and let $0\leqslant s\leqslant (n-1)/4$. Then, modulo $\Phi_n(q)^3$,*

$$\begin{aligned}\sum_{k=s}^{(3n-3)/4}&[8k+3]\frac{(q^3;q^4)_{k-s}(q^3;q^4)_{k+s}(q^3;q^4)_k^2}{(q^4;q^4)_{k-s}(q^4;q^4)_{k+s}(q^4;q^4)_k^2}q^{-2k}\\ &\equiv [3n+4s]\frac{(q^3;q^4)_s^2(q^3;q^4)_{(3n-3)/4+s}(q^6;q^4)_{(3n-3)/4-s}}{(q^4;q^4)_s^2(q^4;q^4)_{(3n-3)/4+s}(q^3;q^4)_{(3n-3)/4-s}}q^{s+(9-9n)/4}.\end{aligned} \quad (4)$$

It should be mentioned that the $s=0$ case of (4) was first given by Guo and Schlosser [3, Lemma 2.3]. Moreover, letting $n=p$ be a prime and taking $q\to 1$ in (4), we are led to the following result: for any prime $p\equiv 1 \pmod 4$ and $0\leqslant s\leqslant (p-1)/4$, modulo p^3,

$$\sum_{k=s}^{(3p-3)/4}(8k+3)\frac{(\frac{3}{4})_{k-s}(\frac{3}{4})_{k+s}(\frac{3}{4})_k^2}{(k-s)!(k+s)!k!^2}\equiv (3p+4s)\frac{(\frac{3}{4})_s^2(\frac{3}{4})_{(3p-3)/4+s}(\frac{3}{2})_{(3p-3)/4-s}}{s!^2(1)_{(3p-3)/4+s}(\frac{3}{4})_{(3p-3)/4-s}}.$$

Tang [10] also proved that, for any positive integer $n \equiv 3 \pmod 4$ and nonnegative integer $s \leqslant (n-3)/4$,

$$\sum_{k=s}^{(3n-1)/4} [8k+1] \frac{(q;q^4)_{k-s}(q;q^4)_{k+s}(q;q^4)_k^2}{(q^4;q^4)_{k-s}(q^4;q^4)_{k+s}(q^4;q^4)_k^2} q^{2k}$$
$$\equiv [3n+4s] \frac{(q;q^4)_s^2(q;q^4)_{(3n-1)/4+s}(q^2;q^4)_{(3n-1)/4-s}}{(q^4;q^4)_s^2(q^4;q^4)_{(3n-1)/4+s}(q;q^4)_{(3n-1)/4-s}} q^{3s+(1-3n)/4} \pmod{\Phi_n(q)^3}.$$

Similarly, we shall establish the following q-supercongruence.

Theorem 1.2. *Let $n \equiv 3 \pmod 4$ be a positive integer and let $0 \leqslant s \leqslant (n-3)/4$. Then*

$$\sum_{k=s}^{(n-3)/4} [8k+3] \frac{(q^3;q^4)_k^2(q^3;q^4)_{k-s}(q^3;q^4)_{k+s}}{(q^4;q^4)_k^2(q^4;q^4)_{k-s}(q^4;q^4)_{k+s}} q^{-2k}$$
$$\equiv [n+4s] \frac{(q^3;q^4)_s^2(q^3;q^4)_{(n-3)/4+s}(q^6;q^4)_{(n-3)/4-s}}{(q^4;q^4)_s^2(q^4;q^4)_{(n-3)/4+s}(q^3;q^4)_{(n-3)/4-s}} q^{s-3(n-3)/4} \pmod{\Phi_n(q)^3}. \quad (5)$$

Likewise, letting $n = p$ be a prime and taking $q \to 1$ in (5), we arrive at the following supercongruence: for any prime $p \equiv 3 \pmod 4$ and $0 \leqslant s \leqslant (p-3)/4$,

$$\sum_{k=s}^{(p-3)/4} (8k+3) \frac{(\frac{3}{4})_{k-s}(\frac{3}{4})_{k+s}(\frac{3}{4})_k^2}{(k-s)!(k+s)!k!^2} \equiv (p+4s) \frac{(\frac{3}{4})_s^2(\frac{3}{4})_{(p-3)/4+s}(\frac{3}{2})_{(p-3)/4-s}}{s!^2(1)_{(p-3)/4+s}(\frac{3}{4})_{(p-3)/4-s}} \pmod{p^3}.$$

Our proof of Theorems 1.1 and 1.2 will make use of the vigorous method of 'creative microscoping', which was introduced by Guo and Zudilin [4]. Meanwhile, we shall utilize Jackson's ${}_6\phi_5$ summation (see [1, Appendix (II.21)]):

$${}_6\phi_5 \left[\begin{matrix} a,\, qa^{\frac12},\, -qa^{\frac12},\, b,\, c,\, q^{-n} \\ a^{\frac12},\, -a^{\frac12},\, aq/b,\, aq/c,\, aq^{n+1} \end{matrix} ; q,\, \frac{aq^{n+1}}{bc} \right] = \frac{(aq;q)_n(aq/bc;q)_n}{(aq/b;q)_n(aq/c;q)_n}, \quad (6)$$

where the *basic hypergeometric series* ${}_{r+1}\phi_r$ is defined as

$${}_{r+1}\phi_r \left[\begin{matrix} a_1, a_2, \ldots, a_{r+1} \\ b_1, b_2, \ldots, b_r \end{matrix} ; q,\, z \right] = \sum_{k=0}^{\infty} \frac{(a_1;q)_k(a_2;q)_k \cdots (a_{r+1};q)_k}{(q;q)_k(b_1;q)_k \cdots (b_r;q)_k} z^k.$$

2 Proof of Theorem 1.1

We require the following q-congruence, which is due to Guo and Schlosser [3, Lemma 2.1].

Lemma 2.1. *Let d, m and n be positive integers with $m \leqslant n-1$. Let r be an integer satisfying $dm \equiv -r \pmod n$. Then, for $0 \leqslant k \leqslant m$, we have*

$$\frac{(aq^r;q^d)_{m-k}}{(q^d/a;q^d)_{m-k}} \equiv (-a)^{m-2k} \frac{(aq^r;q^d)_k}{(q^d/a;q^d)_k} q^{m(dm-d+2r)/2+(d-r)k} \pmod{\Phi_n(q)}.$$

In order to prove Theorem 1.1, we first present the following q-congruence with a parameter a.

Theorem 2.2. *Let $n\equiv 1 \pmod 4$ be a positive integer. Let a be an indeterminate and $0\leqslant s\leqslant (n-1)/4$. Then, modulo $\Phi_n(q)(1-aq^{3n})(a-q^{3n})$,*

$$\sum_{k=s}^{(3n-3)/4}[8k+3]\frac{(aq^3;q^4)_k(q^3/a;q^4)_k(q^3;q^4)_{k-s}(q^3;q^4)_{k+s}}{(aq^4;q^4)_k(q^4/a;q^4)_k(q^4;q^4)_{k-s}(q^4;q^4)_{k+s}}q^{-2k}$$
$$\equiv [3n+4s]\frac{(aq^3;q^4)_s(q^3/a;q^4)_s(q^3;q^4)_{(3n-3)/4+s}(q^6;q^4)_{(3n-3)/4-s}}{(aq^4;q^4)_s(q^4/a;q^4)_s(q^4;q^4)_{(3n-3)/4+s}(q^3;q^4)_{(3n-3)/4-s}}q^{s+(9-9n)/4}. \quad (7)$$

Proof. For $a=q^{-3n}$ or $a=q^{3n}$, the left-hand side of (7) becomes

$$\sum_{k=s}^{(3n-3)/4}[8k+3]\frac{(q^{3-3n};q^4)_k(q^{3+3n};q^4)_k(q^3;q^4)_{k-s}(q^3;q^4)_{k+s}}{(q^{4-3n};q^4)_k(q^{4+3n};q^4)_k(q^4;q^4)_{k-s}(q^4;q^4)_{k+s}}q^{-2k}$$
$$=\sum_{k=0}^{(3n-3)/4-s}[8k+8s+3]\frac{(q^{3-3n};q^4)_{k+s}(q^{3+3n};q^4)_{k+s}(q^3;q^4)_k(q^3;q^4)_{k+2s}}{(q^{4-3n};q^4)_{k+s}(q^{4+3n};q^4)_{k+s}(q^4;q^4)_k(q^4;q^4)_{k+2s}}q^{-2k-2s}$$
$$=[8s+3]\frac{(q^{3-3n};q^4)_s(q^{3+3n};q^4)_s(q^3;q^4)_{2s}}{(q^{4-3n};q^4)_s(q^{4+3n};q^4)_s(q^4;q^4)_{2s}}q^{-2s}$$
$$\times\,{}_6\phi_5\left[\begin{matrix} q^{3+8s}, & q^{\frac{11}{2}+4s}, & -q^{\frac{11}{2}+4s}, & q^3, & q^{3+3n+4s}, & q^{3-3n+4s} \\ & q^{\frac{3}{2}+4s}, & -q^{\frac{3}{2}+4s}, & q^{4+8s}, & q^{4-3n+4s}, & q^{4+3n+4s}\end{matrix}; q^4, q^{-2}\right]. \quad (8)$$

Letting $q\mapsto q^4$, $a=q^{3+8s}$, $b=q^3$, $c=q^{3+3n+4s}$, and $n\mapsto (3n-3)/4-s$ in (6), we see that the right-hand side of (8) may be written as

$$q^{-2s}[8s+3]\frac{(q^{3-3n};q^4)_s(q^{3+3n};q^4)_s(q^3;q^4)_{2s}(q^{7+8s};q^4)_{(3n-3)/4-s}(q^{1-3n+4s};q^4)_{(3n-3)/4-s}}{(q^{4-3n};q^4)_s(q^{4+3n};q^4)_s(q^4;q^4)_{2s}(q^{4+8s};q^4)_{(3n-3)/4-s}(q^{4-3n+4s};q^4)_{(3n-3)/4-s}}$$
$$=[3n+4s]\frac{(q^{3-3n};q^4)_s(q^{3+3n};q^4)_s(q^3;q^4)_{(3n-3)/4+s}(q^6;q^4)_{(3n-3)/4-s}}{(q^{4-3n};q^4)_s(q^{4+3n};q^4)_s(q^4;q^4)_{(3n-3)/4+s}(q^3;q^4)_{(3n-3)/4-s}}q^{s+(9-9n)/4},$$

which is just the right-hand side of (7) with $a=q^{-3n}$ or $a=q^{3n}$. This proves that (7) holds modulo $1-aq^{3n}$ and $a-q^{3n}$.

Since $n\equiv 1 \pmod 4$, letting $d=4, r=3$ and $m=(3n-3)/4$ in Lemma 2.1, we get

$$\frac{(aq^3;q^4)_{m-k}}{(q^4/a;q^4)_{m-k}}\equiv(-a)^{m-2k}\frac{(aq^3;q^4)_k}{(q^4/a;q^4)_k}q^{m(2m+1)+k} \pmod{\Phi_n(q)} \quad (9)$$

for $0\leqslant k\leqslant m$. Applying this q-congruence, it is routine to check that, for $m=(3n-3)/4$ and $s\leqslant k\leqslant m-s$,

$$[8(m-k)+3]\frac{(aq^3;q^4)_{m-k}(q^3/a;q^4)_{m-k}(q^3;q^4)_{m-k-s}(q^3;q^4)_{m-k+s}}{(aq^4;q^4)_{m-k}(q^4/a;q^4)_{m-k}(q^4;q^4)_{m-k-s}(q^4;q^4)_{m-k+s}}q^{2k-2m}$$
$$\equiv -[8k+3]\frac{(aq^3;q^4)_k(q^3/a;q^4)_k(q^3;q^4)_{k-s}(q^3;q^4)_{k+s}}{(aq^4;q^4)_k(q^4/a;q^4)_k(q^4;q^4)_{k-s}(q^4;q^4)_{k+s}}q^{-2k} \pmod{\Phi_n(q)}. \quad (10)$$

Besides, for $(3n-3)/4-s<k\leqslant(3n-3)/4$, the summand indexed k on the left-hand side of (7) is congruent to 0 modulo $\Phi_n(q)$ because $k+s>(3n-3)/4$ and $(q^3;q^4)_{k+s}$ in the numerator contains the factor $1-q^{3n}$. This implies that the left-hand side of (7) is congruent to 0 modulo $\Phi_n(q)$. In view of

$$[3n+4s](q^3;q^4)_{(3n-3)/4+s}=[3n](q^3;q^4)_{(3n-3)/4}(q^{3n+4};q^4)_s\equiv 0 \pmod{\Phi_n(q)}$$

for $n>1$, we conclude that (7) is also true modulo $\Phi_n(q)$. Observing that $1-aq^n$, $a-q^n$, and $\Phi_n(q)$ are pairwise coprime polynomials, we finish the proof of the theorem. □

Proof of Theorem 1.1. Since $(n,4)=1$, the factors related to a in the denominators on both sides of (7) are coprime with $\Phi_n(q)$ for $a=1$. Moreover, when $a=1$ the polynomial $(1-aq^{3n})(a-q^{3n})=(1-q^{3n})^2$ has the factor $\Phi_n(q)^2$. Hence, setting $a=1$ in (7), we get the desired q-supercongruence (4). □

3 Proof of Theorem 1.2

The proof is analogous to that of Theorem 1.1. We first build the following parametric generalization of Theorem 1.2.

Theorem 3.1. *Let $n\equiv 3 \pmod 4$ be an integer greater than 1. Let a be an indeterminate and $0\leqslant s\leqslant(n-3)/4$. Then, modulo $\Phi_n(q)(1-aq^n)(a-q^n)$,*

$$\sum_{k=s}^{(n-3)/4}[8k+3]\frac{(aq^3;q^4)_k(q^3/a;q^4)_k(q^3;q^4)_{k-s}(q^3;q^4)_{k+s}}{(aq^4;q^4)_k(q^4/a;q^4)_k(q^4;q^4)_{k-s}(q^4;q^4)_{k+s}}q^{-2k}$$
$$\equiv[n+4s]\frac{(aq^3;q^4)_s(q^3/a;q^4)_s(q^3;q^4)_{(n-3)/4+s}(q^6;q^4)_{(n-3)/4-s}}{(aq^4;q^4)_s(q^4/a;q^4)_s(q^4;q^4)_{(n-3)/4+s}(q^3;q^4)_{(n-3)/4-s}}q^{s-3(n-3)/4}. \tag{11}$$

Proof. For $a=q^{-n}$ or $a=q^n$, the left-hand side of (11) is equal to

$$\sum_{k=s}^{(n-3)/4}[8k+3]\frac{(q^{3-n};q^4)_k(q^{3+n};q^4)_k(q^3;q^4)_{k-s}(q^3;q^4)_{k+s}}{(q^{4-n};q^4)_k(q^{4+n};q^4)_k(q^4;q^4)_{k-s}(q^4;q^4)_{k+s}}q^{-2k}$$
$$=[8s+3]\frac{(q^{3-n};q^4)_s(q^{3+n};q^4)_s(q^3;q^4)_{2s}}{(q^{4-n};q^4)_s(q^{4+n};q^4)_s(q^4;q^4)_{2s}}q^{-2s}$$
$$\times\,{}_6\phi_5\left[\begin{matrix}q^{3+8s}, & q^{\frac{11}{2}+4s}, & -q^{\frac{11}{2}+4s}, & q^3, & q^{3+n+4s}, & q^{3-n+4s}\\ & q^{\frac{3}{2}+4s}, & -q^{\frac{3}{2}+4s}, & q^{4+8s}, & q^{4-n+4s}, & q^{4+n+4s}\end{matrix};q^4,q^{-2}\right]. \tag{12}$$

Letting $q\mapsto q^4$, $a=q^{3+8s}$, $b=q^3$, $c=q^{3+n+4s}$, and $n\mapsto(n-3)/4-s$ in (6), we see that the right-hand side of (12) has the following closed form:

$$q^{-2s}[8s+3]\frac{(q^{3-n};q^4)_s(q^{3+n};q^4)_s(q^3;q^4)_{2s}(q^{7+8s};q^4)_{(n-3)/4-s}(q^{1-n+4s};q^4)_{(n-3)/4-s}}{(q^{4-n};q^4)_s(q^{4+n};q^4)_s(q^4;q^4)_{2s}(q^{4+8s};q^4)_{(n-3)/4-s}(q^{4-n+4s};q^4)_{(n-3)/4-s}}$$
$$=[n+4s]\frac{(q^{3-n};q^4)_s(q^{3+n};q^4)_s}{(q^{4-n};q^4)_s(q^{4+n};q^4)_s}\frac{(q^3;q^4)_{(n-3)/4+s}(q^6;q^4)_{(n-3)/4-s}}{(q^4;q^4)_{(n-3)/4+s}(q^3;q^4)_{(n-3)/4-s}}q^{s-3(n-3)/4}.$$

This means that (11) is true modulo $1 - aq^n$ and $a - q^n$.

Since $n \equiv 3 \pmod 4$, taking $d = 4, r = 3$ and $m = (n-3)/4$ in Lemma 2.1, we have (9) for $0 \leqslant k \leqslant m$ once more. Using this q-congruence, we can easily verify (10) for $m = (n-3)/4$ and $s \leqslant k \leqslant m - s$. Moreover, for $(n-3)/4 - s < k \leqslant (n-3)/4$, the summand indexed k on the left-hand side of (11) is congruent to 0 modulo $\Phi_n(q)$ because $k + s > (n-3)/4$ and $(q^3;q^4)_{k+s}$ in the numerator incorporates the factor $1 - q^n$. This indicates that the left-hand side of (11) is congruent to 0 modulo $\Phi_n(q)$. In light of

$$[n+4s](q^3;q^4)_{(n-3)/4+s} = [n](q^3;q^4)_{(n-3)/4}(q^{n+4};q^4)_s \equiv 0 \pmod{\Phi_n(q)}$$

for $n > 1$, we conclude that (11) also holds modulo $\Phi_n(q)$. □

Proof of Theorem 1.2. For $a = 1$ the polynomial $(1 - aq^n)(a - q^n) = (1 - q^n)^2$ has the factor $\Phi_n(q)^2$. Thus, taking $a = 1$ in (11), we obtain the q-supercongruence (5). □

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